Wavelet Analysis of Backscattering Data from an Open-Ended Waveguide Cavity

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Abstract— **The wavelet analysis technique is applied to analyze the electromagnetic backscattering data from an open-ended waveguide cavity. Compared to the conventional short-time Fourier transform, the wavelet transform results in a better representation of the scattering features in the time-frequency plane, due to its multiresolution property.**

I. Introduction

THE electromagnetic energy backscattered from an unknown target can provide information useful for classifying and identifying the target. This is commonly accomplished by interpreting the radar echo in either the time or the frequency domain. For target characteristics which are not immediately apparent in either the time or the frequency domain, the joint time-frequency representation of the radar echo can often provide more insight into the scattering mechanisms and, when properly interpreted, can lead to successful identification of the target. In a recent paper by Moghaddar and Walton [1], the joint time-frequency analysis of an open-ended waveguide cavity was carried out. In that work, both the short-time Fourier transform (STFT) and the Wigner–Ville distribution were applied to the broad-band backscattering data from a dispersive cavity structure to arrive at the time-frequency representation. Good insights on the scattering mechanisms were gained from the results. However, the STFT is limited by its fixed resolution in both the time and the frequency domain. The Wigner–Ville distribution, although providing good localization of scattering mechanisms, introduces additional cross terms which leads to "ghosts" in the time-frequency plane.

The theory of wavelets is currently attracting a great deal of attention in many disciplines of applied science [2]–[5]. In this letter, the wavelet transform is applied to the backscattering data from an open-ended waveguide cavity in order to derive the time-frequency representation of the signal. Contrary to the conventional STFT that has fixed resolution in both time and frequency, the wavelet transform, when properly defined, can provide variable resolution in time and multiresolution in frequency. Since the early-time radar echo from finite objects usually consists of sharp peaks [6], very fine time resolution is needed to resolve the various scattering centers. On the other hand, since the late-time arrivals are characterized by resonant ringing, good frequency resolution (or coarse time resolution) is needed for isolating the target resonances. In the intermediate region, dispersive phenomena require good resolution in both time and frequency. The multiple resolution property of the wavelet transform is ideally suited for this task. Consequently, the wavelet representation can provide a better time-frequency characterization of the backscattering data.

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II. Short-Time Fourier Transform and Wavelet Transform

The conventional STFT of a time signal $f(t)$ is defined as [3], [4]:

$$S(\tau,\Omega)=\int f(t)g(t-\tau)e^{-j\Omega t}dt. \tag{1a}$$

It is essentially the Fourier transform operation with the addition of a time window function $g(t)$. The translation of the window as a function of τ results in a two-dimensional time-frequency representation, $S(\tau,\Omega)$, of the original time function. By manipulating (1a), the STFT can also be expressed in the frequency domain:

$$S(\tau,\Omega)=e^{-j\Omega t}\int F(\omega)G(\Omega-\omega)e^{j\tau\omega}d\omega. \tag{1b}$$

Here $F(\omega)$ and $G(\omega)$ are the Fourier transforms of $f(t)$ and $g(t)$, respectively. We observe from (1a) and (1b) that the STFT representation can be obtained through either a moving window in time $g(t)$ or a corresponding moving window in frequency $G(\omega)$.

We will now introduce the (continuous) wavelet transform of a time signal $f(t)$ for our application:

$$W_f(\tau,\Omega)=\int f(t)\tau^{-1/2}h(t/\tau)e^{j\Omega\tau}dt. \tag{2a}$$

By comparing (1a) and (2a), we see that $h(t)$ is similar to the window function $g(t)$ in the STFT. However, $h(t)$ must satisfy an additional "admissibility condition" in wavelet theory [5], viz., $h(t=0)=0$. To satisfy this condition, $h(t)$ is usually chosen to be a translated window function with its center at t_0. By changing τ, the center of the window function moves as τt_0 and the width of the window is dilated by the scale factor τ. The ratio between the window width and the window center (or the Q-factor of the window function) remains fixed for all time. This is in contrast to the STFT where the window width is fixed for all time. By properly manipulating (2a), the wavelet transform can also be carried out on the Fourier transform $F(\omega)$ of the original time signal:

$$W_f(\tau,\Omega)=\int F(\omega)\tau^{1/2}H(\tau(\omega-\Omega))d\omega. \tag{2b}$$

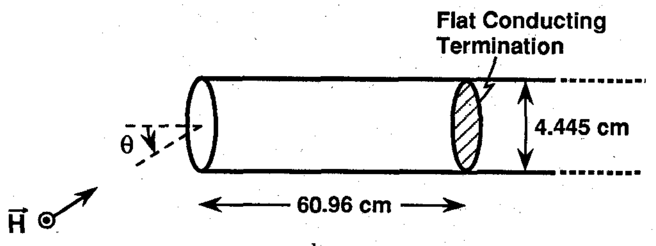


Fig. 1. Geometry of the open-ended circular waveguide cavity.

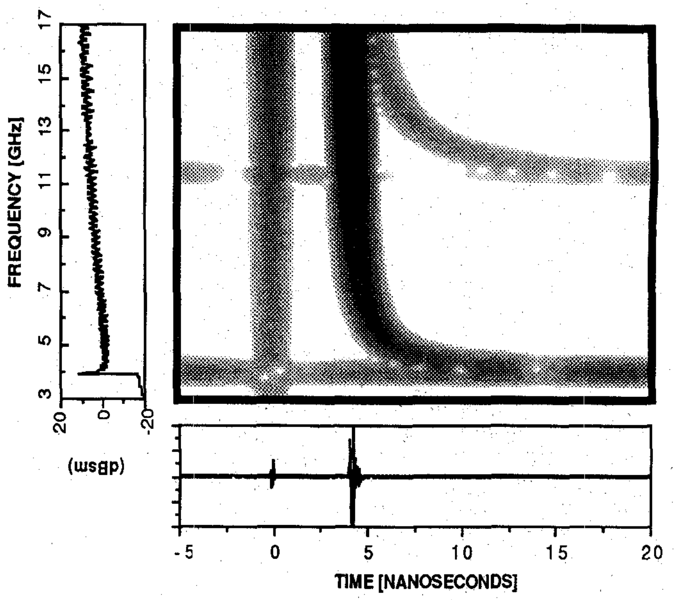
$H(\omega)$ is the Fourier transform of $h(t)$ and is usually referred to as the "mother wavelet." The operation in (2b) can be interpreted as the decomposition of the frequency signal $F(\omega)$ into a family of shifted and dilated wavelets $H(\tau(\omega - \Omega))$. It is important to point out here that the present definition of the wavelet transform in its time and frequency forms is exactly opposite to the common definition of the wavelet transform used in time-series signal analysis [5].

III. Time-Frequency Representation of Backscattering Data

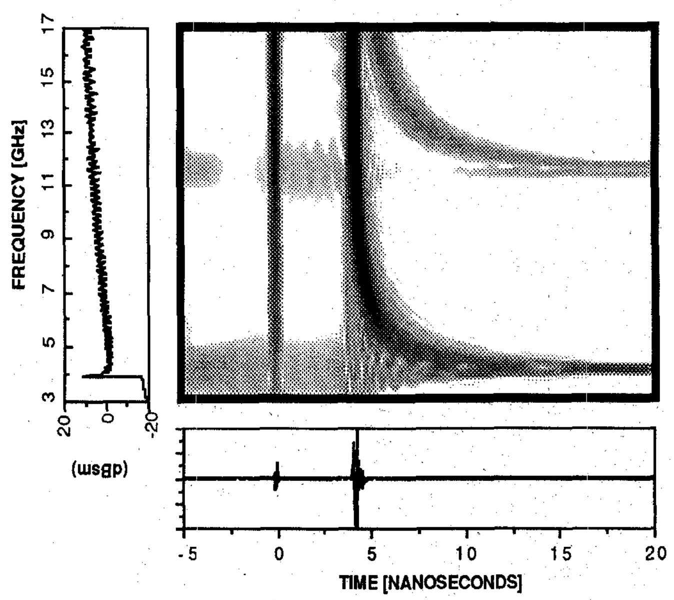
The time-frequency representation of the backscattering data from an open-ended cavity is considered. The cavity is an open-ended circular waveguide with a diameter of 4.445 cm. A flat conducting termination exists 60.96 cm inside the waveguide (Fig. 1). To generate the backscattering data, the radar cross section of this target is first computed in the frequency domain. We take into account of the interior cavity contribution using a modal approach [7] and the diffraction contribution from the front rim of the cavity using the asymptotic formula in [8]. It has been previously established that the backscattering data predicted in this manner agree reasonably well with experiments. The time-domain response is then obtained by Fourier transforming the band-limited frequency data (from 2 to 18 GHz).

Fig. 2(a) shows the time-frequency plot of the backscattering data at normal incidence ($\theta = 0°$) using the STFT. In performing the STFT, a 2-GHz Kaiser–Bessel window in the frequency domain is used in equation (1b). Also plotted along the two axes are the time-domain and the frequency-domain responses. It is apparent that the scattering features are much better resolved in the time-frequency domain than in either the time or the frequency domain alone. Both the nondispersive rim diffraction and the two mode spectra due to the TE_{11} (with cutoff at 3.96 GHz) and the TE_{12} (11.45 GHz) mode can be clearly identified. The mode spectra are in fact dispersion curves of the waveguide modes since the phase velocity of each mode is proportional to the travel time. The noncausal noise appearing in the time-frequency plot is caused by the modal approximation used in simulating the backscattering data. If actual measurement data were used, this noise should be absent.

Due to the fixed resolution of the STFT, the scattering features in Fig. 2(a) are smeared out in the time-frequency plane. This problem is overcome by using the wavelet transform that provides a much better representation of the scattering features in the time-frequency plane, as shown in Fig. 2(b). The wavelet



(a)



(b)

Fig. 2. Time-frequency representation of backscattering data from an open-ended cavity under normal incidence. The grayscale plots of intensity are in decibels with a dynamic range of 40 dB. (a) Short-time Fourier transform (STFT) representation. (b) Wavelet transform representation.

transform is implemented using equation (2a) with the aid of the FFT. The function $h(t)$ is chosen to be a two-sided Kaiser–Bessel window with a Q-factor of 0.3. The $t = 0$ reference of $h(t)$ is located midway between the time events from the rim diffraction and interior contribution (at $\tau = 2.05$ ns). The variable time resolution of the wavelet transform allows sharper time resolution to be achieved during the early-time response and sharper frequency resolution (coarser time resolution) to be achieved during the late-time response. Thus, wavelet transform provides good resolution in identifying the scattering centers and resolving the resonant phenomena of the target while adequately describing the dispersive scattering

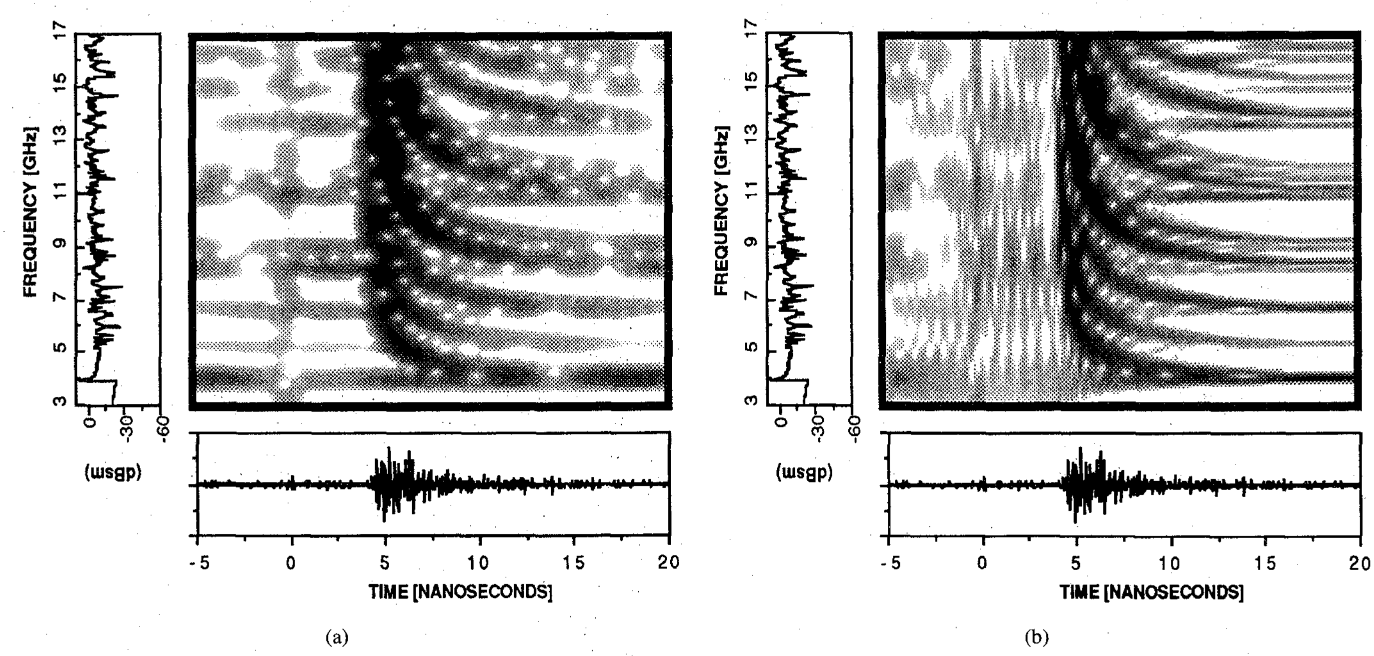


Fig. 3. Time-frequency representation of backscattering data from an open-ended cavity under 45° incidence. The grayscale plots of intensity are in decibels with a dynamic range of 40 dB. (a) Short-time Fourier transform (STFT) representation. (b) Wavelet transform representation.

mechanisms in the intermediate-time region. Figs. 3(a) and 3(b) show the time-frequency plots, generated using the STFT and the wavelet transform, respectively, of the same cavity at 45° incidence. Many more modes are excited by the obliquely incident wave. Consequently the time domain response is much more dispersive. By comparing Figs. 3(a) and 3(b), we find that the wavelet representation again provides a much sharper resolution of the different scattering mechanisms than the STFT.

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